Abstract—It has been observed that tall protected buildings are sometimes struck by lightning on the corners and nearby edges and sharp features. This can cause considerable damage. In this paper, a 3D model to simulate the inception of upward leader from grounded structures under the influence of lightning stepped leaders is proposed. Our physical model takes into account the real nature of the protected area. The influence of the edges and the corners of the structure on the lightning protection system is investigated. The purpose of our model is to present numerical predictions in order to ensure an efficient lightning protection system.

Index Terms— Attractive radius, earthed building protection, electric field modelling, lightning protection, upward leaders competition.

I. INTRODUCTION

Direct stroke causes accidents such like fire when lightning strikes a building or a specified zone. To protect a structure against direct lightning strokes, a preferred impact point is selected to reduce risk of striking the surrounding structure and conduct the flow of the electric current towards the ground, with minimal impedance on the path followed by lightning. This is provided by lightning conductor systems.

Up to now, the location of the vulnerable points to be struck by lightning on structures and the placement of air terminals are often designed with the rolling sphere method (RSM), which is based on the electrogeometrical model (EGM) [1]. The EGM states that the striking distance is a function of the prospective peak stroke current. As this model disregards the physical basis of the upward leader inception process, propagation parameter of the downward and upward leaders, the importance of the electrical and geometrical properties of the structure, it remains deficient. In particular for high buildings that are not considered in the standards when the rolling sphere radius is smaller than the structure height and where several striking points may occur. However, the knowledge in the domain of the physics of lightning has been improved, especially through the experiments in large air intervals [2], so it seems reasonable to refine this highly empirical model by a more physical approach.

Numerical modelling of lightning attachment should consider the physical conditions of formation and propagation of upward leaders emitted from a Lightning Protection System (LPS). But nothing is advised in the standards to accomplish this.

Our model is based on physical phenomena leading to the formation and the development of positive upward discharges under the influence of their own field and the field produced by the negative downward leader [3][4]. Some results for rods on a flat ground have been presented at ICLP’2004. It was demonstrated that it is possible to use it instead the Electrogeometrical Model especially for tall rods [4].

In this paper, our 3D model has been applied to rods on elevated grounded structures. The purpose is to present numerical predictions to guarantee a more reliable lightning protection system. The influence of the edges and the corners of an earthed structure on the LPS is investigated.

Using simple assumptions respectively on the conditions for upward leader inception and downward-upward leader junction [4][5][6][7], the attractive radius of the corners and the edges are computed as a function of the return stroke current, the lightning rod height, the structure features and the downward-upward leader velocities ratio \( R_v = \frac{v_d}{v_u} \). The computation can be made for any incoming downward leader trajectory (oblique or vertical). It also exhibits the competition between upward leaders. Indeed, at ICLP’2004, we showed that when several rods are installed on a flat ground, the launch of an upward leader at the tip of one of them strongly influences the conditions for leader inception at the other rod tips [5]. The same phenomenon
is observed on other vulnerable structure points. It applies for the placement of roof or edge conductors needed to reduce the probability of side flashing.

II. PRESENTATION OF THE PROPOSED MODEL

Our model will use physical considerations on the lightning interception. It relies on the knowledge of the discharges processes in long air gaps and on the analysis of experimental results obtained with natural or artificial lightning.

The ground is considered as a flat conducting surface. As our model is a leader progression model (LPM), the negative and positive leaders are modelled by a succession of charged segments [3][4][5].

To simulate the propagation of the downward and upward leaders towards each other, it is necessary to consider the velocity ratio $R_v$, i.e. the ratio of the downward leader velocity to the upward leader's. Generally, this ratio is arbitrary fixed to one. However, this ratio should still unknown. So we can only make some assumptions. First, in reality, this ratio should not be treated as a constant. Indeed, the upward leader propagates in an increasing electric field, due to the approach of the downward leader. As a consequence, assuming that the downward leader velocity is nearly constant, the upward leader velocity should increase during the propagation and the velocity ratio should decrease. To take this into account, we chose for $R_v$ a range of values from 0.5 to 4 [4][5].

A. Downward and upward leader charge distribution

Each leader is simulated by a linear charge distribution $\lambda$ and a corona charge $Q$ at the tip [4][5][6] (fig. 1). These distributions are established on the basis of the streamer extension zone, more precisely over the minimal length of the streamer resulting from the top of a point, at the moment of the birth of the leader. The choice of a minimal length brings to a minimal charge for the ascending leader, what will give a bigger reliability of the protection.

Let us study the formation of a streamer corona at the top of a rod plunged in an electric field during the approach of a downward leader. The average length of the streamer filaments is $L_s$. This extends of the top of the rod until a distance where the intensity of the electric field reaches a value of 5 kV/cm for a positive streamer (or 11 kV/cm for a negative streamer) [7][8]. By approximating the streamer zone to a half-sphere [6], the following equation is found:

$$E_s = \frac{Q}{2\pi \varepsilon_0 L_s^2}$$  (1)

where $E_s$ is in V/m, $Q$ is the distributed charge in the streamer zone which develops at the top of the point in Coulombs and $L_s$ is the half sphere radius in metres.

When the streamer zone advances one step, the charge $Q$ is distributed on a $L_s$ (m) segment length. If $\lambda$ (C/m) is the linear distribution of the load along this leader segment, the equation will be:

$$E_s = \frac{\lambda L_s}{2\pi \varepsilon_0 L_s^2} = \frac{\lambda}{2\pi \varepsilon_0 L_s}$$  (2)

The studies on air gap discharges in positive polarity evaluate this minimal charge $\lambda_{min}=50\mu$C/m. The value of the corona charge $Q_c$ is deduced from the equation (1):

$$Q_c = \lambda L_s = 2\pi \varepsilon_0 L_s E_s = \lambda \left(\frac{2\pi \varepsilon_0 E_s}{2\pi \varepsilon_0 L_s}\right)$$  (3)

These values of the linear charge density and the corona charge are constant during the propagation in this model.

For the downward negative leader, $\lambda_d$ (C/m) is related to the return stroke current $I$ (A) by the equation [6]:

$$\lambda_d = 0.43 \times 10^6 I^{0.65}$$  (4)

Relation (3) and (4) lead to the following expression for the downward leader corona charge $Q_d$:

$$Q_d = 3.33 \times 10^9 I^{0.3}$$  (5)

Both linear charge density and corona charge are maintained constant throughout the propagation [4][5][8].

B. Electric field of the downward and upward leaders

With the structure present, the electric field and the potential are computed, by solving the Laplace’s equation in the 3D volume between the cloud bottom and the ground surface. The finite element method was chosen with providing boundaries of potential over the problem region and a well fitted mesh refinement on the edges and the corners of the structure. The region where the electric field intensity is high is divided into smaller and finer tetrahedral elements (fig. 2).

A successful capture by a Franklin rod needs three conditions to be fulfilled:
- Formation of an upward leader at the tip of the Franklin rod;
- Stable propagation of the upward leader (from the rod) towards the downward leader;
- Final junction between the two leaders.
Practical experience has shown that sometimes lightning strikes tall structures on the edges or on the corners or on the sides or on structural features at a lower level [2][9][10]. Figure 3 shows an example of expected strikes to the side of a tower as a function of its height. It can be seen that above 100 m there is a marked increase in the probability of stroke [10]. If the structure is provided with LPS, the first theoretical reason is that it is protected according to one level and only flashes with low currents can strike. Another physical reasons and more realistic are firstly due to the trajectory of the downward leader which is not vertical as it is assumed in the lightning protection models. Furthermore, significant change of the field repartition between the cloud and the ground introduced by the tall structure and the ambient electric field intensification on this striking parts. To reduce the probability of strikes to structures, we have at first to take all the structural geometric parameters into account in our calculation and to use not only vertical trajectory, but also oblique.

In order to be more restrictive, some assumptions were made about the determination of the onset electric field for the formation of an upward leader. The electric field was calculated at different fixed points at 1m from the rod tip, and all the corners and edges. When its value at one of these points is greater than or equal 5 kV/cm, there will be a formation of upward leader.

At each propagation step, the electric field and the potential are calculated at fixed distance from the leader tips over various directions. The next step is then directed along the maximum mean field line [5].

Between each downward and upward leaders propagation step, the electric field, along the straight line which joins the upward and downward tips leaders, is calculated and compared to the minimal propagation electric field of the positive streamer (5 kV/cm at atmospheric pressure). As the upward leader propagates towards the downward leader, this minimalist approach was used. When all the computed values are greater than or equal to 5kV/cm, we consider that the conditions for the junction between the upward and downward leaders are fulfilled. If the upward leader was born from the rod, this is leading to a successful interception of lightning. In the opposite case, if it is one which arises from a corner or edge, the attachment of lightning occurs with the structure, leading to a failure of the lightning protection system [5] [8].

### III. RESULTS AND DISCUSSION

During a thunderstorm, the ambient electric field reaches high values. It appears an intensification of the electric field on a protected structure not only on the lightning rod but also on its corners, edges and all the structural features. These places on the structure will be then vulnerable points. The modelling results which follow indicate that these vulnerable points are much more likely struck.

In figure 4, we show the distribution of the electric field on the roof (a) and on a frontage (b) of a protected structure during a thunderstorm. We notice that besides the lightning rod, obviously the corners and the edges are involved in this phenomenon. So, we must take into account in our calculation all these points where there is high electric field intensification.
Firstly, we wanted to know how behaves a corner of a building in the presence of downward leader. It is possible to demonstrate that there will be connection between an ascending leader and a corner by using our model.

Let us consider a simple structure without LPS. In figures 5 and 6, examples of the calculation of the attractive radius of a corner and an edge are given according to the geometry of a rectangular building (the height \( H \) and the width \( W \)) and the lightning current intensity \( I \) and the velocity ratio \( R_v \). Note that there is a dependence on structure width as well as height. Hence, more the structure is high and slender, more corners are more vulnerable that the edges.

Hence, these results are of most practical interest in the field of lightning protection and confirm those found by other researchers [9] [10] [11].

Consequently, a rectangular structure presents an attractive zone as it is illustrated on the figure below. This “vulnerable” area must be considered during the lightning protection process.

![Fig. 7. Upper sight of the attractive area of rectangular structure](image)

In the second issue, the influence of the edges and the corners of the structure on the LPS are investigated. The electrogeometrical model is the only reference used in the international standards, but it does not consider the existence of corners and edges. So, we wanted to verify if a protection built on the basis of this model is really reliable.

If the rod was set at the middle of a flat ground squared surface of width \( W=40 \text{ m} \), for a lightning current of \( I=10 \text{ kA} \), the electrogeometrical model would lead to a height of a Franklin rod of \( h=11 \text{ m} \). As it is shown in figure 8, the lateral distance \( D \) offered by this rod is supposed to be sufficient to protect this flat surface.

![Fig. 8. An illustration of the protected area by using the electrogeometrical model](image)
Now we consider that this squared surface, with the same lightning rod, is not on the ground but on a structure of height $H=100$ m (roof of building). The simulation, using our model, shows that downward leader does not always strike the lightning rod. So, the structure is not well protected. Figures 9 and 10 give examples respectively of a corner and an edge strike. It also exhibits the competition between upward leaders. Indeed, at ICLP'2004 [5], we showed that when several rods are installed on a flat ground, the launch of an upward leader at the tip of one of them strongly influences the conditions for leader inception at the other rod tips. The same phenomenon is observed on other vulnerable structure points.

Another interesting situation will be found according to the height $h$ of the rod. Figure 11 illustrates an example of the successful protection when a 20 m high Franklin rod is used.

There is a common observation of bad reliability of short air terminations—placement at a considerable distance from the “vulnerable” points of the structures. Field observations show that if the height difference between the air termination and the structure is small, the positioning of the air termination is much more critical. Rezinkina [13] used the so called “electrostatic factor” method. She defined the ratio between the maximum level of electric field strength on object $E_s$ and on lightning rod $E_r$ upon electrical field (fig. 10), similar to one that precedes to a lightning discharge:

$$k_d = \frac{E_r}{E_s}$$ (6)

and if $k_d > 2.0$, the objects at the nominated distance $d$ from the air termination (of a height $h$) are protected (fig. 12).
Our model is used to verify this criterion. On figures 13 and 14 we display two computed results. These figures show that despite $kd=3.1$, we have junction with the edge or the corner and a failure of the protection.

![Diagram](image1)

**Fig. 13.** An example of a competition between 2 upward leaders. Junction with the corner. $I=10\,\text{kA}$, $R_v=2$, $kd=3.1$, $d=1\,\text{m}$, $h=1\,\text{m}$.

![Diagram](image2)

**Fig. 14.** An example of a competition between 2 upward leaders. Junction with the edges. $I=10\,\text{kA}$, $R_v=2$, $kd=3.1$, $d=1\,\text{m}$, $h=1\,\text{m}$.

**IV. CONCLUSION**

In this paper a 3D numerical model is proposed in order to improve the lightning protection of elevated structures. The numerical model is based upon the physical phenomena leading to the formation and the development of a positive upward leader in the field produced by the negative downward leader charge and by some other competing upward leaders. This successful analysis applies to the placement of roof or edge conductors needed to reduce the probability of side flashing.

The 3D numerical computations show that when we want to protect a building, we have to take into account all the parts of this structure where the electric field intensity can be high. Indeed, we demonstrated that for example a corner or an edge can attract the lightning.

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The 3D numerical computations show that when we want to protect a building, we have to take into account all the parts of this structure where the electric field intensity can be high. Indeed, we demonstrated that for example a corner or an edge can attract the lightning.

Our propagation model can also be use with complex structure and take into account the environment of the studied configuration.

Another possible application of our model is the real case of an incoming oblique downward leader (no more vertical). This initial trajectory of the negative downward leader will affect the protection area offered by a unique rod.

Our model confirms the model on which the international standards are based and additionally exhibits the competition between upward leaders when these several potential lightning striking points on the structure are taken into account. That explains the observed striking at some high protected structures.

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**VI. REFERENCES**